

SNAKE GAME FOR MEMORIZATION OF MULTIPLICATION

Material

A box containing colored bead bars for the numerals one to nine. There should be eight to ten bars for each number.

One - red

Two - green

Three - pink

Four - yellow

Five - blue

Six - lavender

Seven- white

Eight - brown

Nine - dark blue

A box with a set of black and white bead bars. All the bead bars through five are black. From six through nine the bead bars are black for the first five beads and white for successive beads. This allows for ease in identifying the quantity on the bead bar.

A box of ten bead bars

A notched strip of cardboard

A box of golden bead bars

Presentation

1. Remove the black and white bead bars and place them to the upper right of the work area in graded order.
2. Place the box of golden bead bars in the upper left of the work area.
3. Select an arrangement of colored bead bars and lay them “snake-wise” across

the work area. Use only three or four kinds of bead bars and use several of each.

4. Proceed as in the addition snake game to add the colored bead bars and change them to golden ten bars.
5. Verification: "Let's see if I have added correctly." Line up the ten bars and any remaining black and white beads to the left, and line up the colored bead bars to the right.
6. Bring down a set of colored bead bars that are the same. Multiply them to determine the total. Match an equal amount of golden bead bars to the colored bead bars. Exchange golden bead bars when necessary to obtain the correct quantity to match to the colored bead bars.
7. Continue as above for each set of colored bead bars that are the same.
8. When all the colored bead bars have been placed in one-to-one correspondence to the golden bead bars, verification is complete.

Aim

Direct

Development of order, concentration, coordination, independence and exactness.

To memorize the addition facts.

To make the relationship between addition and multiplication.

To show that multiplication is faster than repeated addition.

Age

5 to 6 years.

CUBING CHAINS FOR THE MEMORIZATION OF MULTIPLICATION

Material

Wooden bead cabinet with top ledge to hold the cubes from left to right: ten cube, nine cube . . . one cube. Hooks extending down from the top ledge of the cabinet to hold the cubing chains from left to right: ten cubing chain, nine cubing chain . . . one cubing chain.

From the right side of the cabinet, ledges extend out in increasing lengths so that the tenth ledge forms the bottom border of the bead cabinet. The top ledge on the right contains the one square and one squaring chain. And each ledge contains the next number in sequence down to the bottom ledge, which contains the ten ten-squares and the ten squaring chain.

Labels organized by cubing chains.

Presentation 1

1. Bring a single bead, a three bead-bar, the three squaring chain, and the three cube.
2. Lay out the single bead and say, "This is a bead."
3. Lay out the three bead-bar and say, "This is a bead bar."
4. Lay out the three squaring chain and say, "This is a squaring chain."
5. Lay out the three bead-square and say, "This is a bead square."
6. Lay out the three cubing chain and say, "This is a cubing chain."
7. Lay out the three bead-cube and say, "This is a bead cube."
8. With the three-period lesson and discussion, repeat the nomenclature.

Aim

Direct

Development of order, concentration, coordination, independence and exactness.

To memorize the multiplication facts.

Age

5 to 6 years.

Presentation 2

1. Select a squaring chain. Usually the four, five, or six is best for illustration of the procedure.
2. Carry the chain with great care, one hand at each end of the chain.
3. Stretch the chain stretched out vertically.
4. Place the square at the top of the chain.
5. With care and dexterity, move one bar up at a time to form the square.
6. Superimpose the bead square over the square formed by the chain.
7. Replace the square at the top of the squared chain.
8. Stretch the chain out again.
9. Place the arrows in order to the right of the bead chain.
10. Count the beads, beginning at the top and placing the appropriate arrow by the bead at the end of each set.
11. After placing the final arrow, which is the square of the bead chain, place the bead square to the right of the arrow indicating the square.
12. Walking along the chain, count in sequence from the first set to the last set of bead bars.
13. Walk back counting backwards.

Aim

Direct

Development of order, concentration, coordination, independence and exactness.

To memorize the multiplication facts.

Age

5 to 6 years.

Presentation 3

1. Select a cubing chain. Usually the four, five, or six is best for illustration of the procedure.
2. Remove the chain from the cabinet by removing each loop from the top hook and placing it in the dominant hand. Grasp the other end with the subdominant hand if bead quantity permits, or drape the beads across the subdominant arm.
3. Carry the chain with great care.
4. Stretch the chain stretched out vertically.
5. Remove the squares of the chain, one at a time, from the bead cabinet and place them in a stack at the top of the chain.
6. Remove the cube of the chain and place it to the right of the stacked squares.
7. With great care and dexterity, move one bar of the chain up at a time to form a square.
8. Superimpose the bead square over the square formed by the chain.
9. Place the square to the right of the square formed by the bead bars.
10. Continue in this manner through all the bars of the chain, forming a square and superimposing a bead square, then placing the bead square to the right. In

this way, two long rectangles are formed: one of bead bars and one of bead squares.

11. “How many bead squares do we have?”
12. Beginning with the last bead square, stack the squares while counting. When the cube is formed with the squares say, “(x) squares equal a cube.”
13. With care and dexterity, stretch the cubing chain out.
14. Place the arrows in order.
15. Count the beads, beginning at the top and placing the appropriate arrow at the bead. Continue until the end of the chain.
16. Walking along the chain, read the labels in sequence.
17. Walk back and read the labels backwards.

Aim

Direct

Development of order, concentration, coordination, independence and exactness.

To learn the numbers in linear counting as they are arranged in subsets.

Indirect

To memorize the multiplication facts through skip-counting.

Age

5 to 6 years.

Presentation 4

1. Both the squaring chain and cubing chain for a particular number are layed out.
2. The labels for both chains are placed into position.

3. The two chains are compared.

Aim

Direct

Development of order, concentration, coordination, independence and exactness.

To learn the numbers in linear counting as they are arranged in subsets.

To memorize the multiplication facts through skip-counting.

Age

5 to 6 years.

Presentation 5 (Geometric Forms)

1. Lay all ten bead-chains out.
2. Begin with the one chain, and say, "This is a point."
3. Lay out the two chain, form an angle, and say, "This is an angle."
4. Lay out the three chain, form a triangle, and say, "This is a triangle."
5. Lay out the four chain, form a square, and say, "This is a square."
6. Lay out the five chain, form a pentagon, and say, "This is a pentagon."
7. Lay out the six chain, form a hexagon, and say, "This is a hexagon."
8. Lay out the seven chain, form a heptagon, and say, "This is a heptagon."
9. Lay out the eight chain, form an octagon, and say, "This is an octagon."
10. Lay out the nine chain, form a nonagon, and say, "This is a nonagon."
11. Lay out the ten chain, form a decagon, and say, "This is a decagon."

Aim

Direct

Development of order, concentration, coordination, independence and exactness.

To learn the numbers in linear counting as they are arranged in subsets.

Indirect

To memorize the multiplication facts through skip-counting.

Age

5 to 6 years.

BEAD BARS FOR THE MEMORIZATION OF MULTIPLICATION

Material

A box with ten compartments containing twenty bead bars in each compartment 1 - 10.

Labels with x and = signs.

Presentation 1 (Multiplication with One Multiplicand)

1. Select a bead bar, such as four.
2. Multiply four by all the numbers from 1 to 10.
3. Lay one bar of four horizontally, saying, “1 x 4 = 4,” and place the product of four vertically below the horizontal four.
4. Lay out two bars of four horizontally to the right of the last beads, saying, “2 x 4 = 8,” and place the product of eight vertically below the horizontal fours.
5. Continue in this manner to work all the equations up through 10 x 4 = ____.
6. Each equation may be recorded in the math notebook.
7. Labels may be made to place at the top of each bead equation:
$$1 \times 4 = 4, 2 \times 4 = 8, \text{ etc.}$$
8. Continue to make all the tables.
9. Upon completion of the tables, the Multiplication Control Chart 1 is used to compare to the answers in the math notebook for accuracy.

Aim

Direct

Development of order, concentration, coordination, independence and exactness.

To memorize the multiplication facts.

Age

6 to 7 years.

Presentation 2 (Multiplication by Ten)

1. Select ten of any of the bead bars and line them up horizontally, one under the other.
2. Place the product in golden bead bars vertically under the bead bars.
3. Record the complete equation in the math notebook.
4. Continue as above with each number 1 through 10.
5. Upon completion of these multiples of 10, the Multiplication Control Chart 1 is used to compare to the answers in the math notebook for accuracy.

NOTE:

The child may notice that multiplying by 10 is easiest, because a zero need only be added to the multiplicand. If this realization is not independently arrived at, point it out to the child after the child has had adequate experience with the material.

Aim

Direct

Development of order, concentration, coordination, independence and exactness.

To memorize the multiplication facts.

Age

6 to 7 years.

Presentation 3 (All Factors for One Product)

1. Select a product, such as 12.
2. Lay out twelve beads: one ten-bar and a colored two-bar laid vertically.
3. Begin with the one; see if it can be a factor in the equation. Can one be a factor? Yes, 12 sets of 1 is 12.
4. Lay out the two-bars. Two-bars taken six times equal twelve. The two-bars are laid to the right, horizontally.
5. Lay out the three-bars. Three-bars taken four times equal twelve. The three-bars are laid to the right, horizontally.
6. Continue with the rest of the bead bars and determine which bead bars are factors of the product 12.
7. Upon completion of the factors of a particular product, record them in the math notebook. The Multiplication Control Chart 1 is used to compare to the answers in the math notebook for accuracy.
8. Select another product, and beginning with the one bead bars determine the possible factors 1 through 10.

NOTE:

Some products have many factors. Some products have few factors. Some products have no factors within the range of 1 through 10.

Aim

Direct

Development of order, concentration, coordination, independence and exactness.

To memorize the multiplication facts.

Age

6 to 8 years.

Presentation 4 (Small Multiplication)

1. Select several bars of any number, but no more than ten.
2. Lay out one of the bars and record the equation and product.
3. Lay out a second bead bar and record the new equation and product formed.
4. Lay out the rest of the bead bars, one at a time, and record the equation in the math notebook.
5. The Multiplication Control Chart 1 is used to compare to the answers in the math notebook for accuracy.
6. Continue in this manner, selecting several bead bars and creating an abbreviated multiplication table.
7. Prepare tickets for the children to use. (5 sets of 6), (4 sets of 7), etc.

Aim

Direct

Development of order, concentration, coordination, independence and exactness.

To memorize the multiplication facts.

Age

6 to 8 years.

Presentation 5 (Reverse Factors of the Same Product)

1. Select several bead bars of any number. (5×7)
2. Lay out the bead bars and record the equation in the math notebook.

3. Select the reverse of the equation in bead bars (7 x 5) and lay them to the right of the first bead bars.
4. Record the second equation in the math notebook.
5. Continue in this manner with many selections and inversions.
6. Upon completion of the inverse equations, the Multiplication Control Chart 1 is used to compare to the answers in the math notebook for accuracy.

Aim

Direct

To memorize the multiplication facts.

Age

6 to 8 years.

Presentation 6 (Square of Numbers)

1. Form the square of each number, placing the necessary number of bars.
2. Record the equations in the math notebook.
3. Upon completion of the squares of the numbers, the Multiplication Control Chart 1 is used to compare to the answers in the math notebook for accuracy.
4. Ask the child what figure is formed. Place a square of the number on the square formed by the bead bars.

NOTE:

The square products appear along the diagonal in the Control Chart.

Aim

Direct

Development of order, concentration, coordination, independence and exactness.

To memorize the multiplication facts.

Age

6 to 8 years.

Presentation 7 (Binomial by One Number)

1. Select a binomial. $(4 + 6)$
2. Explain that you wish to multiply $(4 + 6)$ by 4.
3. Place the multiplier and the x sign before the binomial and the = sign after the binomial: $4 \times (4 + 6) = \underline{\quad}$.
4. Record the equation in the math notebook.
5. Multiply the first term by four (4×4), select the bead bars, and lay them out.
6. Multiply the second term by four (4×6), select the bead bars, and lay them to the right of the first bead bars.
7. Place a + sign between them and an = sign after them.
8. Record the second part of the equation: $(4 \times 4) + (4 \times 6) = \underline{\quad}$.
9. Calculate the sum and record it in the math notebook.

$$16 + 24 = 20$$

10. The full write-up in the math notebook is:

$$4 \times (4 + 6) =$$

$$(4 \times 4) + (4 \times 6) =$$

$$16 + 24 = 40$$

11. Select a prepared equation slip and continue as before.

Aim

Direct

Development of order, concentration, coordination, independence and exactness.

To memorize the multiplication facts.

To introduce algebraic formulations with multiplication.

Age

7 to 8 years.

Presentation 8 (Square of a Binomial)

1. Select a hundred square and name what it is: $10 \times 10 = 100$.
2. Record this equation in the math notebook.
3. Divide the square across the top into two parts, for example, $7 + 3$.
4. Count across seven and place a rubber band vertically down the hundred square.
5. Count down seven and place a rubber band horizontally across the hundred square.
6. Indicate the seven square formed on the hundred square. Take the real square of seven and place it over the “mock” square of seven.
7. Indicate the three square formed on the hundred square. Take the real square of three and place it over the “mock” square of three.
8. Indicate the other two shapes that are formed: 3×7 on the right side of the square, and 7×3 on the bottom side of the square.
9. Place the real squares aside.
10. With the bead bars, reproduce the square of seven and the square of three, and place them in the same position as in the real square.

11. Make the rectangles 3 x 7 and 7 x 3 and place them in their relative position next to the squares. A new square has been formed.

12. Record this equation in the math notebook:

$$(7 \times 7) + (3 \times 3) + (3 \times 7) + (7 \times 3) = 100$$

$$49 + 9 + 21 + 21 = 100$$

13. Explain that you wish to rewrite the equation using a special sign. To indicate the square of a number, a 2 high off the line is written.

$$7^2 + 3^2 + (3 \times 7) + (7 \times 3) = 10^2$$

$$49 + 9 + 21 + 21 = 10^2$$

The raised two is called an 'exponent'.

14. Explain that when the same factors are multiplied there is an easier way to record it. Instead of:

$$(3 \times 7) + (7 \times 3)$$

it may be recorded:

$$2(3 \times 7)$$

15. With this information we may record the equation:

$$7^2 + 3^2 + 2(7 \times 3) = 10^2$$

16. Continue with the possible binomials:

$$9^2 + 1^2 + 2(9 \times 1) = 10^2$$

$$8^2 + 2^2 + 2(8 \times 2) = 10^2$$

$$6^2 + 4^2 + 2(6 \times 4) = 10^2$$

$$5^2 + 5^2 + 5^2 + 5^2 = 10^2$$

Direct Aim: Development of order, concentration, coordination, independence and exactness.

To memorize the multiplication facts.

To introduce algebraic formulations with multiplication.

Age

7 to 8 years.

Presentation 9 (Square of the Trinomial)

1. Select a hundred square and, with rubber bands, make a binomial as in the past presentation.
2. Explain that the binomial just completed divides the hundred square into two parts along each side. This produces four areas within the square.
3. Now we wish to divide the hundred square into three parts along each side. This produces nine areas within the square.
4. Divide it along the sides: $5 + 3 + 2$.
5. Create these same areas with the three squares and the bead bars.
6. Analyze the figure moving across the square from left to right:

$$(5 \times 5) + (5 \times 3) + (5 \times 2)$$

$$25 + 15 + 10 = 50$$

$$(3 \times 5) + (3 \times 3) + (3 \times 2)$$

$$15 + 9 + 6 = 30$$

$$(2 \times 5) + (2 \times 3) + (2 \times 2)$$

$$10 + 6 + 4 = 20$$

100 Total

NOTE:

At a later date and after much practice with the squares of the binomial and trinomial, the mathematical expression is introduced.

Lay out the bead squares and bead bars. They mathematically express this:

$$(5 + 3 + 2) \times (5 + 3 + 2) =$$
$$3^2 + 2(3 \times 5) + 2(3 \times 2) + 5^2 + 2(5 \times 2) + 2^2 =$$

Recall that in constructing the square of a trinomial, the mathematical expression assumed nine operations and all nine operations were expressed. The new mathematical expression only requires six operations.

Binomial: Two terms and four operations.

Trinomial: Three terms and nine operations.

Quatrenomial: Four terms and sixteen operations.

Direct Aim:

Development of order, concentration, coordination, independence and exactness.

To memorize the multiplication facts.

To introduce algebraic formulations with multiplication.

Age:

7 to 8 years.

Presentation 10 (Passing from One Square to a Consecutive Square)

1. Select a square: six.
2. Say, "Let's go from this square of six to a square of seven."
3. The square of 6 is 36. The square of 7 is 49. To go from 36 to 49, thirteen beads are required. There is not a bead bar of thirteen to wrap around the six square to create a seven square.
4. Select two 7-bead bars, and place them along the sides of the six

square, and show how they are too long.

5. Select two six bars and place them along the sides of the six square.

With the addition of a single one-bead in the corner, the 7 square is complete.

6. Count the number of beads added to the six square: 13.

7. Verbalize the process: “To go from 6 squared to 7 squared, 2 six-bead bars are added, plus one.

8. Record this in the math notebook: 6^2 to $7^2 = 6^2 + 6 + 6 + 1$

9. The child may then begin with one and pass successively from one square to the next and record it in the math notebook:

$$1^2 \text{ to } 2^2 = 1^2 + 1 + 1 + 1 = 1^2 + 3 = 4$$

$$2^2 \text{ to } 3^2 = 2^2 + 2 + 2 + 1 = 2^2 + 5 = 9$$

$$3^2 \text{ to } 4^2 = 3^2 + 3 + 3 + 1 = 3^2 + 7 = 16$$

$$4^2 \text{ to } 5^2 = 4^2 + 4 + 4 + 1 = 4^2 + 9 = 25$$

$$5^2 \text{ to } 6^2 = 5^2 + 5 + 5 + 1 = 5^2 + 11 = 36$$

$$6^2 \text{ to } 7^2 = 6^2 + 6 + 6 + 1 = 6^2 + 13 = 49$$

$$7^2 \text{ to } 8^2 = 7^2 + 7 + 7 + 1 = 7^2 + 15 = 64$$

$$8^2 \text{ to } 9^2 = 8^2 + 8 + 8 + 1 = 8^2 + 17 = 81$$

$$9^2 \text{ to } 10^2 = 9^2 + 9 + 9 + 1 = 9^2 + 19 = 100$$

NOTE:

In each passage from one square to the next, the amount added increases by two.

Presentation 11 (Passage from One Square to a Non-Consecutive Square)

1. Select a square: six.

2. Say, “Let’s go from this square of six to a square of eight.
3. The square of 6 is 36. The square of 8 is 64. To go from 36 to 64, twenty-eight beads are required. There is not a bead bar of twenty-eight to wrap around the six square to create an eight square.
4. Select four six-bead bars and place two bead-bars along each of two adjacent sides of the square.
5. Where the bead bars intersect is a two square. Select the two square and place it in the space.
6. Count the number of beads added to the six square: 28.
7. Verbalize the process: “To go from 6 squared to 8 squared, 4 six-bead bars are added plus a two square.
8. Record this in the math notebook:

$$6^2 \text{ to } 8^2 = 6^2 + 2(6 \times 2) + 2^2 =$$
$$36 + 24 + 4 = 64$$

9. The child may select squares and make them into non-consecutive squares.

CONSTRUCTION OF THE DECANOMIAL

Material

A box with ten compartments containing fifty-five bead bars in each compartment 1 - 10.

Presentation 1 (Vertical Construction)

1. Say, "Let's construct the multiplication tables with the bead bars in a special way."
2. Lay out one bar for each of the numbers 1 - 10 across the top of the work area.
3. Say, "Let's construct the table of one with the one-beads down the left column."
4. Continue with the table of two with the two-bead bars down the second column.
5. Continue with the rest of the columns.
6. Upon completion of the bead bar, lay out a chart on which to duplicate the pattern.

NOTE:

Multiplicand remains the same and the multiplier changes.

Aim

Direct

Development of order, concentration, coordination, independence and exactness.

To point out the interrelationship of the multiplication facts.

Age

6 to 8 years.